1026. Proposed by Elias Lampakis, Kiparissia, Greece.

Let a, b, c be positive real numbers. Prove that

$$\frac{2a^2-ab-b^2}{a+b}+\frac{2b^2-bc-c^2}{b+c}+\frac{2c^2-ca-a^2}{c+a} \geq \frac{a^2+b^2+c^2-ab-bc-ca}{a+b+c}.$$

Solution by Arkady Alt , San Jose ,California, USA.

Since

$\frac{2a^2-ab-b^2}{a+b} = \frac{2a^2}{a+b}-b$ then $\sum_{cyc}\frac{2a^2-ab-b^2}{a+b} \geq \frac{a^2+b^2+c^2-ab-bc-ca}{a+b+c} \Leftrightarrow$

$\sum_{cyc}\frac{2a^2}{a+b} \geq \frac{a^2+b^2+c^2-ab-bc-ca}{a+b+c}+a+b+c \Leftrightarrow \sum_{cyc}\frac{2a^2}{a+b} \geq$

$\frac{2(a^2+b^2+c^2)+ab+bc+ca}{a+b+c} \Leftrightarrow (a+b+c)\sum_{cyc}\frac{2a^2}{a+b} \geq 2(a^2+b^2+c^2)+ab+bc+ca \Leftrightarrow$

$\sum_{cyc}\frac{2a^2c}{a+b}+\sum_{cyc}\frac{2a^2(a+b)}{a+b} \geq 2(a^2+b^2+c^2)+ab+bc+ca \Leftrightarrow 2\sum_{cyc}\frac{c^2a^2}{ca+bc} \geq ab+bc+ca$

and by Cauchy Inequality $\sum_{cyc}(ca+bc)\cdot\sum_{cyc}\frac{c^2a^2}{ca+bc} \geq \left(\sum_{cyc}ca\right)^2 \Leftrightarrow$

$2\sum_{cyc}ca\sum_{cyc}\frac{c^2a^2}{ca+bc} \geq \left(\sum_{cyc}ca\right)^2 \Leftrightarrow 2\sum_{cyc}\frac{c^2a^2}{ca+bc} \geq \sum_{cyc}ca.$